VBB VIAK

APPENDIX 2
EEP19/82

ICE FORCE STUDIES IN THE BOTHNIAN BAY Phase 2

PROCESSING OF RESPONSE DATA DURING PHASE 1

VBB VIAK R 10210 A

ENZ/Iceproj14/ap-fs.18

PROCESSING OF RESPONSE DATA DURING PHASE 1

1. General

Three alternative procedures were developed during Phase 1 for the analysis of recorded accelerograms, as described in this Appendix. The methods are, in principle, applicable to high-frequency as well as low-frequency signals. Hence, the general notations are used instead of dashed symbols indicating low-frequent signals, although the procedures are primarily applied only to these, since the high-frequent signals can be processed with simpler procedures, as described in Chapter 4.

The recorded acceleration is composed of two components, the true horizontal acceleration $\ddot{x}$ and the "tilt acceleration $\ddot{x}_\phi$ due to the inclination of the structure and the attached inclinometer as explained in the description of the instruments, Appendix 1. (1)

$$\ddot{x}_R = \ddot{x} - \ddot{x}_\phi$$

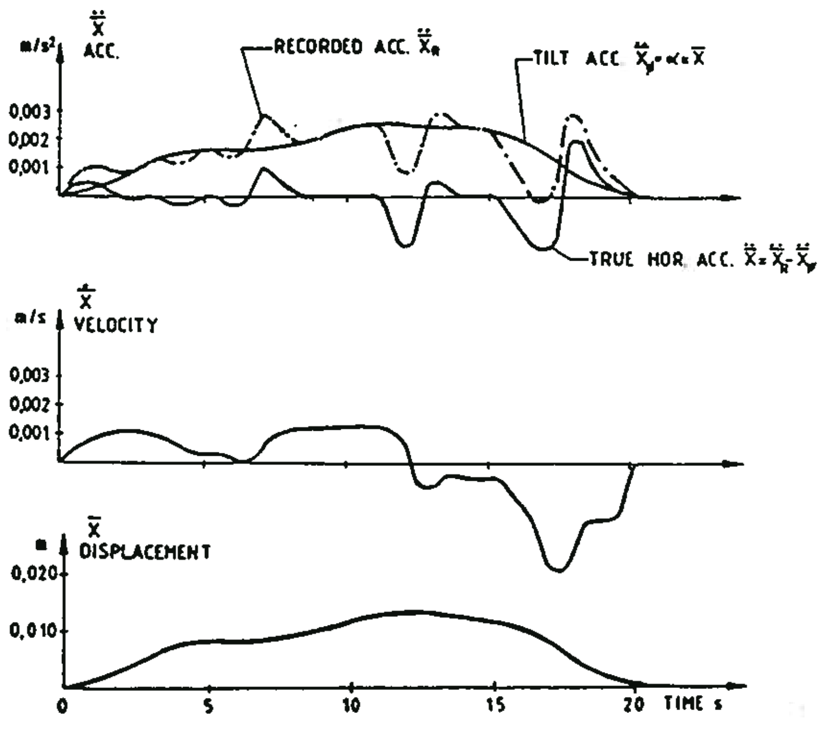

Figure A2:1

Separation of true horizontal and tilt components of measured accelerations

The three alternative procedures developed during Phase 1 are described in Section 2, 3 and 4, respectively. The first method was based on stepwise integration of the accelerograms, in order to determine the deflection of the structure and thereby identify the tilt angle. Like other methods based on integration, the accuracy of the results depends strongly on the resolution of measured quantities which limited the practical use of the method. The second method was developed in order to avoid this sensitivity of the resolution. However, when Fourier transformation was introduced, the particular relationships between acceleration and deflection of a sinusoidal motion time-history could be taken advantage of. The separation of the horizontal and gravity components of the measured acceleration could be made in the frequency domain with a very good accuracy.

Therefore, the third method, described in Section 4 of this appendix, was preferred already during the end of Phase 1 and has been used without exception during Phase 2.

2. Separation of tilt and true acceleration components by integration of recorded accelerations at one level

The "tilt acceleration" is proportional to the true displacement x.

$$\ddot{x}_\phi = \alpha \cdot x \qquad (2)$$

Consequently the true horizontal acceleration can be expressed

$$\ddot{x} = \ddot{x}_R + \ddot{x}_\phi = \ddot{x}_R + \alpha \cdot x \qquad (3)$$

The true velocity $\dot{x}$ and displacement x can thus be computed by a stepwise integration of

$$\ddot{x}_R + \alpha \cdot x$$

For the time increment $\Delta t = t_{i+1} - t_i$ the following quantities are known:

$\ddot{x}_{Ri}$, $\ddot{x}_{Ri+1}$, $\dot{x}_i$ and x_i

The true velocity can be expressed:

$$\dot{x}_{i+1} = \dot{x}_i + \Delta\dot{x} = \dot{x}_i + \ddot{x}_{mean} \cdot \Delta t = \dot{x}_i + \frac{\ddot{x}_i + \ddot{x}_{i+1}}{2} \cdot \Delta t =$$

$$= \dot{x}_i + \frac{\ddot{x}_{Ri} + \ddot{x}_{Ri+1}}{2} \cdot \Delta t + \alpha \cdot \frac{x_i + x_{i+1}}{2} \cdot \Delta t \qquad (4)$$

The true displacement can be expressed:

$$x_{i+1} = x_i + \Delta x = x_i + \dot{x}_{mean} \cdot \Delta t =$$

$$= x_i + \frac{\dot{x}_i + \dot{x}_{i+1}}{2} \cdot \Delta t =$$

$$= \dot{x}_i + \dot{x}_i \cdot \Delta t + \frac{\ddot{x}_{Ri} + \ddot{x}R_{i+1}}{4} \cdot \Delta t^2 + \alpha \cdot \frac{x_i + x_{i+1}}{4} \cdot \Delta t^2 \qquad (5)$$

i.e.

$$x_{i+1} = \frac{x_i + \dot{x}_i \cdot \Delta t + \frac{\ddot{x}_{Ri} + \ddot{x}_{Ri+1}}{4} \cdot \Delta t^2 + \frac{\alpha \cdot x_i}{4} \cdot \Delta t^2}{1 - \frac{\alpha \cdot \Delta t^2}{4}}$$

$\dot{x}_{i+1}$ is obtained from Eq. (4) and

$\ddot{x}_{i+1}$ from Eq. (3)

The tilt angle

$$\phi_i = \frac{\ddot{x}_{\phi i}}{g} = \frac{\ddot{x}_i - \ddot{x}_{Ri}}{g} \qquad (7)$$

The static ice force

$$F = \frac{K\phi}{Z_F} \cdot \phi \qquad (8)$$

where K_ϕ is obtained from the static mode of the "transfer model's" deflection.

According to Eq. (2):

$$\alpha = \frac{\ddot{x}_\phi}{x} = \frac{\phi \cdot g}{x} \qquad (9)$$

This factor varies with the level referred to and with the mode of deflection (static or dynamic). Therefore the "static" inclination of the structure must be evaluated after separating the "static" response function from the "dynamic".

For the two accelerometer levels at NORSTRÖMSGRUND and for static loads the following values can be used:

$$\alpha_1 = \alpha_2 = \frac{0.784 \cdot 10^{-3} g}{0.032} = 0.240$$

$$\alpha_3 = \alpha_4 = \frac{0.784 \cdot 10^{-3} g}{0.015} = 0.513$$

The accuracy of these preliminary α-values have been checked by test loading of the structures.

One implication of the integration procedure described above is that the accuracy of the result is extremely dependent on the accuracy of the initial boundary condition as well as the recorded acceleration values introduced during the integration process. Since errors increase exponentially with time, the process should only be applied to limited time intervals, starting from boundary conditions where the tilt or the horizontal acceleration can be assessed with a very good accuracy. Such boundary conditions may be identified and quantified using the procedure described in the following section.

3. Determination of the inclination from acceleration records at two levels

The tilt component $\ddot{x}_\phi$ of the recorded acceleration $\ddot{x}_R$ can be separated from the true horizontal acceleration $\ddot{x}$ by taking advantage of simultaneous acceleration measurements at the upper and lower points of measurements and the following relations:

From Eq. 1:

$$\ddot{x}_1 = \ddot{x}_{R1} + \ddot{x}_{\phi 1} \qquad (10)$$

$$\ddot{x}_3 = \ddot{x}_{R3} + \ddot{x}_{\phi 3} \qquad (11)$$

where the indices 1 and 3 refer to the upper and lower acceleration values, respectively, measured in the same horizontal direction. (The perpendicular components shall of course be accounted for in the same manner).

Provided that the lighthouse structure behaves elastically, the ratio between the upper and lower horizontal displacements is constant and so is the corresponding ratios between the true horizontal acceleration values:

$$x_1/x_3 = \ddot{x}_1/\ddot{x}_3 = \text{Const.} \qquad (12)$$

Furthermore, since the inertia contributions are negligible for the low-frequency modes of deflections, the tilt angle is essentially the same for the upper as for the lower measurement point, both points being located above the level of ice action:

$$\ddot{x}_{\phi 1} = \ddot{x}_{\phi 3} = \ddot{x}_\phi \qquad (13)$$

The tilt component $\ddot{x}_\phi = \phi \cdot g$ can thus be derived from the following set of equations:

$$\ddot{x}_1 = \ddot{x}_{R1} + \ddot{x}_\phi \tag{14}$$

$$\ddot{x}_3 = \ddot{x}_{R3} + \ddot{x}_\phi \tag{15}$$

$$\ddot{x}_1/\ddot{x}_3 = x_1/x_3 = \text{Const.} \tag{16}$$

As a result the tilt acceleration can be expressed:

$$\ddot{x}_\phi = \phi \cdot g = \frac{1}{x_1/x_3-1} \cdot \ddot{x}_{R1} - \frac{x_1/x_3}{x_1/x_3-1} \cdot \ddot{x}_{R3} \tag{17}$$

Finally the static ice force components are obtained using Eq. 8.

The ratios x_1/x_3 and x_2/x_4 are obtained from the transfer model. For the NORSTRÖMSGRUND transfer model the preliminary numerical value is:

$x_1/x_3 = x_2/x_4 = 2.133$

(Since $\alpha = \frac{\phi \cdot g}{x}$ and $\phi_1 = \phi_3$ the ratios

$\frac{\alpha_3}{\alpha_1} = \frac{\alpha_4}{\alpha_2} =$ have the same value as x_1/x_3 etc.)

The following expression for the tilt angle at NORSTRÖMSGRUND was obtained:

$$\phi = \frac{1}{g} (0.883 \cdot \ddot{x}_{R1} - 1.883 \cdot \ddot{x}_{R3}) \tag{18}$$

It is obvious that, if the horizontal acceleration is large compared with the "tilt acceleration",the tilt will be derived as a small difference between large numbers, which may impair the accuracy significantly. The tilt angle should therefore preferably be derived for situations where the true horizontal acceleration is comparatively small. This is indicated, for instance, by the appearance of sequences of <u>linear</u> time functions of the recorded acceleration $\ddot{x}_R$, particularly such sequences where the acceleration is constant. Under most circumstances, this indicates that the horizontal acceleration is close to zero and that the true deflection velocity is zero or constant, i.e. the ice force and the spring forces of the structure are in equilibrium.

This also justifies the assumption that the "static" mode of deflection is predominant.

4. Determination of the inclination by transformation of accelerograms into the frequency domain

A recorded acceleration time-history $\ddot{x}_R(t)$ can be represented by Fourier component sinusoids. The amplitude $\ddot{x}_{Rf}$ of a component wave representing the frequency interval fi±Δf/2 in the recorded accelerogram represents the difference between the true horizontal acceleration component $\ddot{x}_f$ and the tilt component $\ddot{x}_\phi$ (See Section 5.2 of the main report):

$$\ddot{x}_{Rf} = \ddot{x}_f - \ddot{x}_{\phi f} = \ddot{x}_f - \alpha_f \cdot x_f \tag{19}$$

Since the component waves are sinusoidal, the relationship between the horizontal displacement x_f and the corresponding acceleration $\ddot{x}_f$ can be expressed:

$$x_f = -\ddot{x}_f/\omega^2 \tag{20}$$

Hence

$$\ddot{x}_{Rf} = \ddot{x}_f + \alpha_f \cdot \frac{\ddot{x}_f}{\omega^2}$$

For each component wave of the recorded accelerogram the true horizontal acceleration and the tilt components can thus be separated:

$$\ddot{x}_f = \frac{\omega^2}{\omega^2 + \alpha f} \cdot \ddot{x}_{Rf} \tag{21}$$

$$\ddot{x}_{\phi f} = \phi \cdot g = \frac{-\alpha_f}{\omega^2 + \alpha_f} \cdot \ddot{x}_{Rf} \tag{22}$$

$$\alpha_f = \frac{\phi \cdot g}{x_f}$$

expresses the relationship between the tilt angle and the horizontal displacement at the instrument level and is frequency dependent. It can be determined from the transfer model.

Example:

- For $f = 1$ Hz; $\alpha_f = 0.5$: $\ddot{x}_{\phi f} = -0.0125\ \ddot{x}_{Rf}$
  $\ddot{x}_f = 0.9875\ \ddot{x}_{Rf}$
- For $f = 0.2$ Hz; $\alpha_f = 0.5$: $\ddot{x}_{\phi f} = -0.285\ \ddot{x}_{Rf}$
  $\ddot{x}_f = 0.715\ \ddot{x}_{Rf}$
- For $f = 0.1$ Hz; $\alpha_f = 0.5$: $\ddot{x}_{\phi f} = -0.559\ \ddot{x}_{Rf}$
  $\ddot{x}_f = 0.441\ \ddot{x}_{Rf}$
- For $f = 0.01$ Hz; $\alpha_f = 0.5$: $\ddot{x}_{\phi f} = -0.9922\ \ddot{x}_{Rf}$
  $\ddot{x}_f = 0.0078\ \ddot{x}_{Rf}$